# Connected Tasks: The Building Blocks of Reasoning and Proof

KERRI RICHARDSON, TYRETTE CARTER & SARAH BERENSON describe how pattern blocks can be used to engage students in a variety of open-ended challenges.

Do you find it challenging to find mathematical tasks that promote reasoning in your classroom? What type of tasks do you feel are the most important for children to investigate? Finding patterns, and making and justifying conjectures are considered the building blocks of mathematical reasoning and proof. Curriculum revisions in the United States and Australia place increased emphasis on problem solving and reasoning in the primary school curriculum (National Council of Teachers of Mathematics [NCTM], 2000; Australian Curriculum, Assessment and Reporting Authority [ACARA], 2010). A number of curriculum resources for teachers are available (e.g., NCTM, 1993; Sullivan & Lilburn, 1997) but under current reform efforts, primary teachers require additional ideas to extend problem solving and reasoning in their classrooms.

We have conducted teaching experiments with Grade 5 students that engaged them in generalising and justifying rules using three pattern-block tasks. The majority of students were able successfully to generalise to find an explicit rule, and to justify their rules. Individually, students in Grades 3 and 4 have also successfully completed the pattern-block tasks. We attribute the success of these students' reasoning activities to the three tasks that shared common features. We have called sets of tasks with common

features, connected tasks. Connected tasks share relationships, contexts, properties, and/or operations. These types of task are one way to encourage reasoning and proof throughout each grade level. The pattern-block tasks we describe here are examples of connected tasks that share relationships and contexts. Other examples of connected tasks are the tower task, pizza problem, and the taxicab problem that share relationships and properties (Maher & Martino, 1996; Powell, Francisco, & Maher, 2003). Figure 1 shows students working with unifix cubes to find patterns.

Figure 1. Finding patterns using unifix cubes.

In this article, we offer explicit ways that pattern-block tasks can be used throughout the primary grades. We ask: which tasks prompt reasoning about patterns, structures, and regularities? The goal of this article is to encourage all mathematics teachers to offer connected tasks as part of the day to day curriculum, and to demonstrate ways in which one task can be used in many ways for various grade levels. Figure 2 is a list of features of connected tasks that we have identified. In the following sections each of these features is discussed along with ways in which pattern block tasks can be used to promote reasoning in the classroom.

**Features of connected tasks that promote reasoning**

- Includes open-ended questions
- Supports reasoning at multiple grade levels
- Promotes predictions and encourages multiple conjectures
- Allows students to answer why
- Requires skillful questioning and listening

Figure 2. Connected task features.

## Includes open-ended tasks

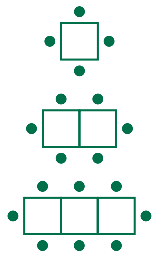
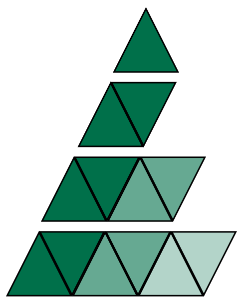
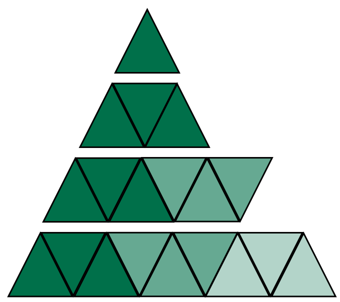
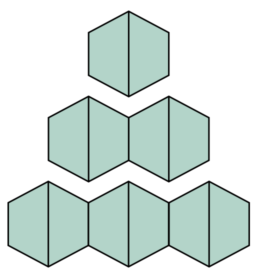
Connected tasks include open-ended questions that require free responses. For example, the question, "What is the next number?" is a closed question only requesting the appropriate number or a single word response. In contrast to this, an open-ended question such as, "What makes this a pattern?" supports students' thinking and allows students to provide evidence. The goal of presenting connected tasks that include open-ended questions is to generate discourse, allow flexibility in thinking, and encourage various solutions at different ability levels. Hence, the types of questions in connected tasks challenge students to reason logically and explain their solutions.

## Supports reasoning at multiple grade levels

Our work uses the same connected tasks in multiple primary grades since these tasks accommodate for a variety of levels of mathematical sophistication. For example, Tasks 2–4 in Table 1 are designed to help younger students think about the growth of patterns and make sense of the properties of triangles and parallelograms such as sizes, sides, and angles. Students explore placing

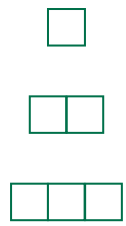
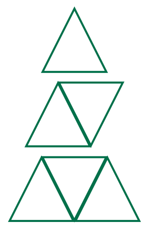
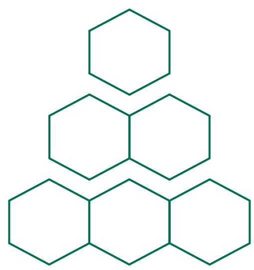
several triangles together to form a new shape. They are given opportunities to identify and explain patterns. The use of color is important when studying parallelograms in relation to the triangles that are used to form them. Furthermore, when asked to predict how many triangles it takes to form three parallelograms students observe the growth of a pattern. The models connect the parallelograms and trapeziums in a similar manner to Task 1. This is aimed at highlighting consistencies and connections between the tasks, thereby preparing young students for more sophisticated growth patterns in the later grades.

Table 1. Connected Tasks Grades K–2 (adapted from Phillips et al., 1991, pp. 49–50).

| Context | K–2 Questions | Models of the sequence |
|---|---|---|
| **Task 1**<br>(squares and students, can also use chairs) | • How many students can you fit around one table if only one student can fit on one side?<br>• How many students can you fit around 2 tables if only 1 student can fit on one side?<br>• 3 tables?<br>• What patterns do you see?<br>• Can you predict how many students can fit around 4 tables that are put together?<br>• How do you know? | <br>Tables \| Students<br>1 \| 4<br>2 \| 6<br>3 \| 8 |
| **Task 2**<br>(two triangles connected together) | • How many triangles make<br>1 parallelogram?<br>2 parallelograms?<br>3 parallelograms?<br>• Can you predict how many triangles it takes to make 4 parallelograms?<br>• How do you know?<br>• What patterns do you see? |  |
| **Task 3** | • How many triangles make<br>1 trapezium*?<br>2 trapeziums?<br>3 trapeziums?<br>• Can you predict how many triangles it takes to make 4 trapeziums?<br>• How do you know?<br>• What patterns do you see? |  |
| **Task 4** | • How many trapeziums in<br>1 hexagon?<br>2 hexagons?<br>3 hexagons?<br>• Can you predict how many trapeziums it takes to make 4 hexagons?<br>• How do you know?<br>• What patterns do you see? |  |

*referring to the isosceles trapezium in row 2 of the pattern

Table 2. Connected Task Grades 3–5 (adapted from Phillips et al., 1991, pp. 49–50).

| | 3–5 Tasks | Models of the sequence |
|---|---|---|
| **Task 1** | • What is the perimeter of a<br>1-block square train?<br>2-block square train?<br>3-block square train?<br>• Can you predict the perimeter of a<br>10-block square train?<br>• How do you know?<br>• What patterns do you see? |  |
| **Task 2** | • What is the perimeter of a<br>1-block triangle train?<br>2-block triangle train?<br>3-block triangle?<br>• Can you predict the perimeter of a<br>10-block triangle train?<br>• How do you know?<br>• What patterns do you see? |  |
| **Task 3** | • What is the perimeter of a 1-block<br>hexagon train?<br>1 hexagon<br>2 hexagons<br>3 hexagons<br>• Can you predict the perimeter of a<br>10-block hexagon train?<br>• How do you know?<br>• What patterns do you see? |  |

Initial questions such as, "How many chairs can you fit around two tables that are put together?" are posed to gather data about the patterns students explore. As they continue finding patterns, it is vital to encourage students to think about how and why the patterns occur.

The connected tasks in Table 2, for Grades 3–5 and beyond, extend students' reasoning with higher levels of mathematical thinking on perimeter.

Through the use of pattern blocks in these tasks, students begin to collect data, name variables, use tables, draw the models, and most importantly write rules. If older students are new to this type of pattern-finding, it is appropriate to begin with the initial questions used in the K–2 tasks. To assist with differentiating and scaffolding the learning at all levels and building student's use of higher levels of reasoning, we have provided a chart in Table 3. When we offered these tasks to Grade 5 students,, we had several who were able to fill out this table to make additional connections between the tasks.

Table 3. Pattern task table for upper primary students.

| | Triangle Train | Square Train | Pentagon Train | Hexagon Train | Decagon Train |
|---|---|---|---|---|---|
| **Rule** | | | | | |
| **Explanation** | | | | | |
| **General rule to find perimeter for any shape train** | | | | | |
| **Explanation of the general rule for any shape train** | | | | | |

**Suppose I wanted to find the perimeter of a 50-block train made out of decagons. Look at the rules you have already found. Can you find a pattern that will help you write a rule for the perimeter of a decagon train with 50 cars?**

## Promotes predictions and encourages multiple conjectures

Connected tasks provide an opportunity for students to make predictions and conjectures about patterns they observe. We discovered that building a model of a particular sequence with the pattern blocks was the first step towards encouraging predictions and conjectures. We asked, “Can you predict how many chairs can fit around four tables joined together?” After building three stages, the answer “10” was readily apparent to students.

Next we asked students to collect and organise their data in a chart or T-table (input/output), naming the variables. After constructing their tables and organising their data, we asked students to predict the number of chairs by asking for the 5th, 10th, and 100th term in the sequence. Finally, our students searched to find patterns leading to a conjecture. We asked, “What is your conjecture [rule] about this pattern?” Students recorded and explained their conjectures to the class on an interactive whiteboard. We found that students make different conjectures because there are several correct conjectures for each task. The use of data from their tables and pattern block models helped students to justify their conjectures. This enhanced students’ abilities to determine which conjectures were valid.

For example, when we posed the square table task to a group of Grade 5 students, there was a heated discussion about 100 tables seating 220 people versus 202 people. The students who thought the correct answer was 220 were positive that the way to arrive at the solution was to complete their T-tables using their patterns until there were 10 connected tables. Since 22 people sit around 10 tables, the students were certain they must take this and multiply by 10 because 10×10 is 100. Therefore, 10×22 gives you 220. The other half of the class insisted that this method did not utilise the rule found, which they called, “multiply by 2 and add 2.” They said to find the people seated at 100 tables, you must multiply 100 by 2 to account for the top and bottom and then add 2 to account for the end pieces, thus giving you 202 as a final answer. The 220 group were convinced that 202 did in fact make more sense.

Connected tasks provide opportunities for students to employ multiple representations, communicate ideas, and associate these ideas with their prior knowledge, often building on their problem solving strategies. Encouraging multiple solutions using connected tasks helps students’ problem solving skills (Leikin & Levav-Waynberg, 2008).

## Allows students to answer why

There are a variety of ways that our students reasoned about these connected tasks. There is no one right answer, but there are incorrect answers. Asking "Why?" helped our students determine what explanations were valid or true. If this is the first time your students use patterns to generalise a rule and provide a justification, they will probably use additive reasoning with a focus on chairs or triangles. For example, they see a recursive pattern of 4, 6, 8 … chairs. A prediction of the 4th term in the sequence can be found by adding two to the last term. This is very appropriate for primary students, and they can usually explain "Why?" their predictions are correct by building a model of the 4th term. At this point their focus may be on the number of chairs, generally disregarding the number of tables. The conjecture that some students make is that you "add two each time you add a table." A good answer to "Why?" is that for each term, their models support their rules.

With more pattern-finding practice, our Grade 5 students were able to move beyond additive reasoning to multiplicative reasoning. Some saw the relationship between the number of tables and the number of chairs. At first the focus of our students was on model building and how the last table added changed the number of chairs. As students learned to focus on the whole model or parts of the whole model, their generalisations became more explicit. We observed that some students saw the two end tables have three chairs while the inner tables only have two chairs. Their rules conveyed the idea that you take the number of tables minus two, and multiply that number by two. Then you add 6 for the two end tables (see Figure 3). The focus on the joined tables is, thus, on the middle and the two ends as separate conditions.

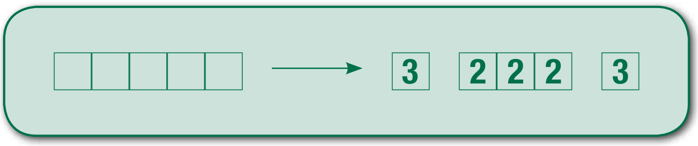



Figure 3. The joined tables and end tables as separate conditions.

Other students seem to be able to focus on the whole model keeping in mind that the two ends have an extra chair. Their focus is directed at the top and bottom of the model, and therefore, they multiply the number of tables by two. Accounting for the end tables, they conjecture that if you multiply the number of tables by two and add two they can use this rule to determine how many chairs can fit around 100 tables (See Figure 4).

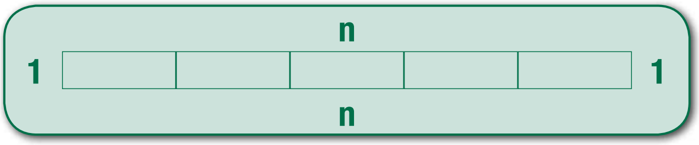



Figure 4. The model as a whole, including top, bottom, and ends.

The first rule can be expressed: $c = 2\,(t - 2) + 6$, where $c$ is the number of chairs and $t$ is the number of tables. Simplifying this rule: $c = 2t - 4 + 6$ or $c = 2t + 2$ is the same rule as in Figure 4. Either one of these rules can be justified convincingly with the pattern block models to answer, "Why?" Figure 5 shows the work on this task of Grade 3 student. Gabe's rule indicates multiplicative reasoning.

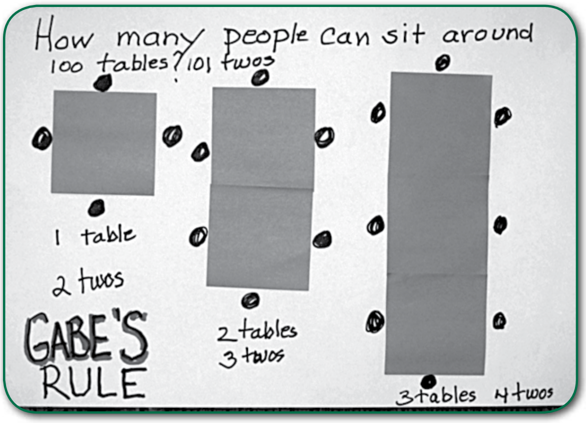



Figure 5. Gabe's work on the square pattern block problem.

## Requires skillful questioning and listening

Establishing a strong mathematics community in the classroom involves careful questioning and listening by both teachers and students. Chazan and Ball (1995) noted that only allowing students to discuss their thinking about a problem with one another fails to

promote a strong mathematics community. The role of the instructor is important not only in facilitating conversations about particular ideas, but also to help students understand misconceptions; as in the "220 versus 202" example discussed earlier.

The lead teacher/researcher was able to bring to the forefront a misunderstanding held by the Grade 5 students. She set the stage for future work with connected tasks once they understood why the "220" solution was not mathematically sound. Stein (2007) argues that for students to be engaged in the mathematics classroom, teachers must provide a safe community where argumentation coupled with conceptual understanding is centered at the heart of instruction. Davis (1997) focuses on the importance of listening to students in a way that is not judging the correctness or otherwise of answers, but instead listening for the ideas leading to and from the solutions.

## Conclusion

Using what was learned with the square pattern blocks in the teaching experiment the Grade 5 students engaged in the triangle and hexagon table tasks (see Tables 1 and 2), connecting what they learned about square table patterns. We suggest that these connected tasks emphasise different solutions and support students' ability to conjecture and argue for or against a particular rule. This enhances the classroom interaction and increases the students' level of sophistication in justifying, explaining, and solving future mathematical problems (Yackel & Cobb, 1996).

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